Superstable triangle.(**MOG variant**)
https://www.linkedin.com/feed/update/urn:li:activity:6644445407814516736
Suppose that we have an triangle with sides a,b,c such that for every positive integer n there exists a triangle with sides a^n, b^n and c^n.
Prove that the triangle must be equilateral.

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We will prove that for triangle ABC with side length a,b,c the numbers a^n, b^n and c^n are side lengths of a triangle for every $n \in \mathbb{N}$ iff ABC is isosceles with equal sides lengths, which not less then third one.

For any real x,y,z let $\Delta(x,y,z) := 2xy+2yz+2zx-x^2-y^2-z^2$. Note that $\Delta(x^2,y^2,z^2) = (x+y+z)(x+y-z)(x-y+z)(-x+y+z)$ and for positive x,y,z we have equivalency $\Delta(x^2,y^2,z^2) > 0 \Leftrightarrow \begin{cases} x+y>z \\ y+z>x \\ z+x>y \end{cases}$.

Thus, a^n, b^n and c^n satisfies triangle inequalities iff $\Delta(a^{2n}, b^{2n}, z^{2n}) > 0$.

Let $n \in \mathbb{N}$ be any and a^n, b^n and c^n be side lengths of a triangle, that is $\Delta(a^{2n}, b^{2n}, z^{2n}) > 0$.
Due symmetry and homogenity of $\Delta(a^{2n}, b^{2n}, c^{2n}) > 0$ WLOG we assume that $a \geq b \geq c$ and $c = 1$.

Then for any $n \in \mathbb{N}$ we have $\begin{cases} \Delta(a^{2n}, b^{2n}, c^{2n}) > 0 \\ a \geq b \geq c = 1 \end{cases} \Leftrightarrow \begin{cases} b^n + 1 > a^n \\ a \geq b \geq 1 \end{cases}$.

Suppose that $a > b$, then by Bernoulli Inequality $a^n = (b+(a-b))^n = b^n\left(1+\dfrac{a-b}{b}\right)^n >$ $b^n\left(1+\dfrac{a-b}{b}\cdot n\right) = b^n + n(a-b)b^{n-1} > b^n + n(a-b) > b^n + 1$ for any $n > \max\left\{\dfrac{1}{a-b}, 1\right\}$.
This contradicts the fact that inequality $b^n + 1 > a^n$ holds for any $n \in \mathbb{N}$.
Thus $a = b$ and, therefore, triangle should be isosceles with two equal sides, which not less then third one.
Let now $a = b \geq c$ then $\Delta(a^{2n}, b^{2n}, c^{2n}) = 2a^nb^n + 2b^nc^n + 2c^na^n - a^{2n} - b^{2n} - c^{2n} =$ $4c^na^n - c^{2n} \geq 3c^n > 0$.